## Chapter 20 - Root (W) Scales

### 20.1 The Form of the W Scales.

The Faber-Castell Slide Rules $2 / 83 \mathrm{~N}$ and $62 / 83$, incorporate these scales. In the case of $2 / 83 \mathrm{~N}$, the W scales are actually 50 cm . "C and D scales" cut in hat at $\sqrt{10}$ (i.e. 3.162) and fitted on a 25 cm ( 10 inch) Slide Rule. For the $62 / 83$ they are 25 cm . "C and D scales" cut in half and fitted on a 12.5 cm ( 5 inch) slide rule. For the $2 / 83 \mathrm{~N}$, the W scale gives approximately four significant figures of accuracy, wile the $62 / 83$ can be read to about three figures, thus making calculations with the W scales on the 12.5 cm ( 5 inch ) Slide Rule as accurate as the normal C and D scales on a 25 cm ( 10 inch) Slide Rule.

### 20.2 Multiplication

The procedure differs somewhat from that used with the C and D scales. Depending on the numbers involved we have to use either the left (1) or right (10) black index marks, or one of the red index marks located on the right hand end of the $\mathrm{W}^{\prime}{ }_{1}$ and $\mathrm{W}_{1}$ scales, or the left hand end of the $\mathrm{W}_{2}$ and $\mathrm{W}^{\prime}{ }_{2}$ scales. The following examples show each of the four possible variations.

Example 1: $1.2 \times 21=25.2$

1. Place the left (black) index of the $\mathrm{W}^{\prime}{ }_{1}$ scale over the 1.2 on the $\mathrm{W}_{1}$ scale.
2. Set the hair line over 21 on the $\mathrm{W}^{\prime}{ }_{1}$ scale.
3. Under the hair line read off 25.2 on the $\mathrm{W}_{1}$ scale as the answer. (Note, under the hair line 6.64 (at 4) on the $\mathrm{W}^{\prime}{ }_{2}$ scale gives $1.2 \times 6.64=7.97$ (at 5) on the $\mathrm{W}_{2}$ scale.)

Example 2: $8.86 \times 19.3=171$

1. Place the right (black) index of the $\mathrm{W}^{\prime}{ }_{2}$ scale over 8.86 on the $\mathrm{W}_{2}$ scale.
2. Set the hair line over 19.3 on the $\mathrm{W}^{\prime}{ }_{1}$ scale.
3. Under the hair line read off 171 on the $\mathrm{W}_{1}$ scale as the answer. (Note, under the hair line 6.1 (at 4 ) on the $\mathrm{W}^{\prime}{ }_{2}$ scale gives $8.86 \times 6.1=54.1$ (at 5 ) on the $\mathrm{W}_{2}$ scale).

Example 3: $2.37 \times 6.85=16.25$

1. Place the right (red) index of the $\mathrm{W}^{\prime}{ }_{1}$ scale over 2.37 on the $\mathrm{W}_{1}$ scale.
2. Set the hair line over 6.85 on the $W^{\prime}{ }_{2}$ scale.
3. Under the hair line read off 16.25 on the $\mathrm{W}_{1}$ scale as the answer.
(Note, under the hair line 21.65 (at 4 ) on the $\mathrm{W}^{\prime}{ }_{1}$ scale gives $2.37 \times 21.65=51.4$ (at 5 ) on the $\mathrm{W}_{2}$ scale.)
Example 4: $39.9 \times 50.6=2020$
4. Place the left (red) index of the $\mathrm{W}^{\prime}{ }_{2}$ scale over 39.9 on the $\mathrm{W}_{2}$ scale.
5. Set the hair line over 50.6 on the $\mathrm{W}^{\prime}{ }_{2}$ scale.
6. Under the hair line read off 2020 on the $\mathrm{W}_{1}$ scale as the answer.
(Note, under the hair line 1.6 (at 4 ) on the $\mathrm{W}^{\prime}{ }_{1}$ scale gives $39.9 \times 1.6=63.9$ ) (at 5 ) on the $\mathrm{W}_{2}$ scale.)
The following table summarizes these procedures for easy reference.

### 20.3 Rules for Division

(i) When the numbers involved in the division are located on adjacent scales, the answer is read off the $\mathrm{W}_{1}$ or $\mathrm{W}_{2}$ scales under either of the black index marks.
(ii) When the numbers involved in the division are located on scales on opposite sides of the rule, the answer is read off the $W_{1}$ or $W_{2}$ scales under either of the red index marks.

## Exercise 20(b)

| (i) | $360 \div 18=$ |
| :--- | :--- |
| (ii) | $4,800 \div 0.6=$ |
| (iii) | $12.25 \div 35=$ |
| (iv) | $43.75 \div 0.0304=$ |
| (v) | $3,025 \div 55=$ |
| (vi) | $1,925 \div 17.5=$ |


| (vii) | $\pi \div 6=$ |
| :--- | :--- |
| (viii) | $93 \div 9,600=$ |
| (ix) | $\frac{219}{17 \times 28}$ |

(x)
$\frac{35}{0.12 \times 0.47}$
$\frac{805}{(104 \times 0.043)}$
(xii)
$1,406 \div 52^{2}=$
$\begin{array}{ll} & \overline{0.12 \times 0.47} \\ \text { (xi) } & \frac{805}{(104 \times 0.043)}\end{array}$

### 20.4 Squares and Square Roots

## A. Squares

For numbers on the $\mathrm{W}^{\prime}{ }_{1}$ and $\mathrm{W}^{\prime}{ }_{2}$ scales, their squares are found on the C scale. We use the $\mathrm{W}^{\prime}{ }_{1}$ and $\mathrm{W}^{\prime}{ }_{2}$ scales in preference to the $\mathrm{W}_{1}$ and $\mathrm{W}_{2}$ scales, as the former are on the slide with the C scale.

Example: $2.46^{2}=6.05$

1. Set the hair line over 2.46 on the $\mathrm{W}^{\prime}{ }_{1}$ scale.
2. Under the hair line read off 6.05 on the C scale as the answer.

Note: In Fig 20.7, under the hair line the 7.78 9at 3 ) on the $\mathrm{W}^{\prime}{ }_{2}$ scale would be read off the scale (a 1 ) as $7.78^{2}=$ 60.5 .

## Exercise 20(c)

| (i) | $7.65^{2}=$ | (v) | $0.084^{2}=$ |
| :--- | :--- | :--- | :--- |
| (ii) | $0.9^{2}=$ | (vi) | $0.00022^{2}=$ |
| (iii) | $65^{2}=$ | (vii) $30.25^{2}=$ |  |
| (iv) $207^{2}=$ | (viii) $\left(5.4 \times 10^{3}\right)^{2}=$ |  |  |

## B. Square Roots

For a number on the C scale, its square root is read off the $\mathrm{W}^{\prime}$, scale if the number is between 1 and 10 , or off the $W^{\prime}{ }_{2}$ scale if the number is between 10 and 100 . For numbers larger than 100 or less than 1 , we use the procedure as outlined in Unit 5 .

Example 1: $\sqrt{13}=3.605$

1. Set the hair line over 13 on the C scale. (as the number is between 10 and 100 we find its square root on the $\mathrm{W}^{\prime}{ }_{2}$ scale.)
2. Under the hair line read off 3.605 on the $\mathrm{W}^{\prime}{ }_{2}$ scale as the answer.

Example 2: $\sqrt{130}=11.4$

1. Set the hair line over 130 on the C scale. (Express $\sqrt{130}=\sqrt{1.3 \times 100}=\sqrt{1.3} \times 10$, thus we find $\sqrt{1.3}$ ).
2. Under the hair line read off 1.14 on the $\mathrm{W}^{\prime}{ }_{1}$ scale as the value for $\sqrt{1.3}$.

$$
\begin{aligned}
\therefore \text { answer } & =1.14 \times 10 \\
& =11.4
\end{aligned}
$$

Example 3: $\sqrt{0.098}=0.313$

1. Set the hair line over 9.8 on the C scale. (Express $\sqrt{0.098}=\sqrt{\frac{9.8}{100}}=\frac{\sqrt{9.8}}{10}$, thus we find $\sqrt{9.8}$ ).
2. Under the hair line read off 3.13 on the $\mathrm{W}^{\prime}{ }_{1}$ Scale as the value for $\sqrt{9.8}$.

$$
\begin{aligned}
\therefore \text { answer } & =\frac{3.13}{10} \\
& =0.313
\end{aligned}
$$

Exercise 20(d)
(i) $\sqrt{6.25}$
(vi) $\sqrt{2,450}$
(ii) $\sqrt{93.5}$
(vii) $\sqrt{0.06}$
(iii) $\sqrt{1.125}$
(viii) $\sqrt{0.143}$
(iv) $\sqrt{324}$
(ix) $\sqrt{0.4}$
(v) $\sqrt{960}$
(x) $\sqrt{0.0025}$
20.5 Miscellaneous Calculations

The following tables list a number of calculations which make use of the W scales, and form a supplement to the table given in 10.3. For squares and square roots the appropriate W scale must be used according to the numbers involved.

| Example | Set the H.L over | Under the H.L Place |  | Reset the H.L over |  | Under the H.L. answer |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $a^{2} b$ | a on W scale | Index of W' scale |  | B on the C scale |  | on D scale. |
| $a b^{2}$ | a D | Index | C | b | W' | D |
| $a b^{4}$ | b W | a | CI | b | W' | D |
| $a^{2} b^{2}$ | a W | Index | W' |  | W' | D |
| $a^{2} b^{2} c$ | a W | c | CI | b | W' | D |
| $\frac{a}{b^{2}}$ | a D | b | W' | Index | W' | D |
| $\frac{a^{2}}{b^{3}}$ | a W | b | W' | b | CI | D |
| $\frac{a^{2}}{b}$ | a W | b | C | Index | C | D |
| $\frac{a^{3}}{b}$ | a W | b | C |  | C | D |
| $\frac{1}{a b^{2}}$ | a DI | b | W' | Index | W' | D |
| $\frac{1}{a^{2} b^{2}}$ | a W | Index | W' |  | W' | DI |
| $\frac{1}{a^{4}}$ | Index W | a | W' | Index | W' | A |
| $\frac{a^{2} b}{c}$ | a W |  | C |  | C | D |
| $\frac{a^{2} b}{c^{2}}$ | a W |  | W' |  | C | D |
| $\frac{a b}{c^{2}}$ | a D |  | W' |  | C | D |
| $a \sqrt{b}$ | a W | Index | W' | b | C | W |
| $\sqrt{a b}$ | a D | Index | C | b | C | W |
| $\sqrt{a b c}$ | a D |  | CI |  | C | W |

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| $a^{2} \sqrt{b}$ | a W | Index | W' | b | B | D |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\sqrt{\frac{a}{b}}$ | a D | b | C | Index | C | W |
| $\frac{\sqrt{a}}{b}$ | a D | b | W' | Index | W' | W |
| $\frac{a}{\sqrt{b}}$ | a W | b | C | Index | W' | W |
| $\sqrt{\frac{a b}{c}}$ | a D | c | C | b | C | W |
| $\sqrt{\frac{a}{b c}}$ | a D | b | C | c | CI | W |
| $\frac{1}{\sqrt{a b}}$ | a DI | Index | CI | b | CI | W |
| $\frac{1}{a \sqrt{b}}$ | Index W | a | W' | b | CI | W |
| $a \sqrt{b c}$ | b D |  | CI | a | W' | W |
| $a \sqrt{\frac{b}{c}}$ | b D | c | C | a | W' | W |
| $\pi r^{2} h$ | r W | h | CI | Index | C | DF |
| $\frac{1}{2} r^{2} \theta$ | $\begin{aligned} & \theta \text { (degrees) ST } \\ & \theta \text { (degrees) } \mathrm{D} \end{aligned}$ | $\begin{aligned} & 2 \\ & 2 \end{aligned}$ | $\begin{aligned} & \mathrm{C} \\ & \mathrm{C} \end{aligned}$ | $\begin{aligned} & \mathrm{r} \\ & \mathrm{r} \end{aligned}$ | $\begin{aligned} & W^{\prime} \\ & W^{\prime} \end{aligned}$ | $\begin{aligned} & \mathrm{D} \\ & \mathrm{D} \end{aligned}$ |

